

Computer Simulations Using Maple

Central Force Motion

Introduction

In this lab we will use Maple to solve motion central force problems. We will be looking at the gravitational force but another example is the electrical force.

Lab

1. If you have mastered the following you can skip to Procedure 2.
 - a. basic math and algebra using Maple
 - b. plotting using Maple, including labeling the axis and a title
 - c. integrating using Maple, including explicit and implicit
 - d. differentiating using Maple, including first and second order

2. Use Maple to solve the following problem.

Start with Newton's second law for a central force

$$\vec{F}_{net} = m\vec{a}$$

And assume you have a particle of mass m moving in the gravitational field (force) of a much larger mass M ($m \ll M$). Rewrite Newton's second law using the gravitational force and spherical coordinates. You should get equation 7.3.4 or 7.3.5 in your text.

Use Maple to solve for the motion with the following initial conditions:

- The mass of the satellite is 1000 kg
- the satellite is 200 km above the surface of the earth
- the radial velocity is zero and the tangential velocity is 2.4×10^4 m/s

Plot the results for one orbit and show that you have a circular orbit.

The mean radius of the earth is 6.37×10^6 m and has a mass of 5.98×10^{24} kg

3. Increase the tangential velocity by 15% and 30% and plot all three orbits on the same plot. Are the other two orbits circular?
4. Junior, try this!
Do procedure 2 again but have Maple use a numerical method to solve the differential equations. Make sure this is working correctly because we will need this for the next couple of procedures.
5. Alpeh says this is too easy
How do the orbits in procedures 2 and 3 change if the force law is

$$F_r(r) = -G \frac{mM}{r^{2.1}}$$

Plot all three orbits.

6. Can you do this Andrew?

For a central force given by

$$F(r) = -K \frac{e^{-br}}{r^2}$$

what values of r are necessary for stable orbits?

7. Atmospheric friction causes a satellite in a low orbit to slowly spiral inward (do you remember Skylab I?). Consider a 1000-kg satellite in a circular orbit 200-km above the Earth's surface. Let the atmospheric friction be

$$\vec{F}_f = -C e^{-b(r-R)} \vec{v}$$

With $C=250$ kg/s and R is Earth's radius. Use Maple to solve for the satellite's motion and plot the orbit until the satellite is 50 km above the surface.