

An Introduction to Lie Algebras through $\mathfrak{sl}(2)$ and $\mathfrak{sl}(3)$

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This talk will be a short introduction to the theory of Lie Algebras. We will focus mainly on $\mathfrak{sl}(2)$ and its actions (representations).

In Mathematics, Lie **algebra** is an Algebraic structure whose main use is in studying geometric objects such as **Lie Groups** and **Differentiable Manifolds**. The history of Lie Algebras goes back to the idea originally proposed by Felix Klein that the geometry of a space is determined by its symmetries. **Sophus Lie's** contribution was to look at these symmetries from an infinitesimal view point (the **Infinitesimal Lie Groups**). This point of view naturally gives rise to the structure we now know as a **Lie Algebra**. **Lie Groups** represent the best-developed theory of continuous symmetry of mathematical objects and structures, which makes them indispensable tools for many parts of contemporary Mathematics, as well as for Modern Theoretical Physics. They provide a natural framework for analyzing the continuous symmetries of Differential Equations (Differential Galois Theory), in much the same way as Permutation Groups are used in Galois Theory for analyzing the discrete symmetries of Algebraic Equations. Example applications of Lie Algebras can be found in Mechanics, Quantum Mechanics and in the Angular Momentum Operators. Another great example is found in the work of Murray Gell-Mann on Elementary Particles.

We will begin this talk with an explicit computational example and gradually move toward the more abstract theory of representations.